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Magnetotransport in superlattices: I. Impurity scattering

P Vasilopoulos and H Hogues
Département de Génie Physique, Ecole Polytechnique, Montréal, Canada H3C 3A7

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Abstract. The DC conductivity of a superlattice, in the presence of a magnetic field normal to its planes, is evaluated when an electric field is applied in the direction of the superlattice or parallel to its planes for elastic impurity scattering. Short-range and long-range potentials are considered. The effective mass difference between wells and barriers and the interlayer tunnelling are taken into account. The period of the Shubnikov–de Haas oscillations changes as the Fermi level passes through the minibands.

1. Introduction

In the past transport in quantum wells and superlattices has received considerable attention. Recently, the integral quantum Hall effect has been observed in superlattices [1], when a magnetic field is applied normal to its planes. A theoretical investigation of the Hall conductivity σ_{yx} has been reported [2] but, to our knowledge, a similar investigation for this situation, of the conductivity components σ_{xx} and σ_{zz} is missing.

In this paper we evaluate the conductivity along the direction of the electric field taken perpendicular (σ_{xx}) or parallel (σ_{zz}) to the superlattice direction z. The magnetic field is taken along the z direction and the dominant scattering centres are assumed to be static screened impurities only within the wells. We also take account of the effective mass difference between wells and barriers and of the interlayer tunnelling which is essential for the component σ_{zz}.

The paper is organised as follows: in § 1 we present the formalism; we then calculate σ_{xx} in § 3, and σ_{zz} in § 4; conclusions and discussion follow in § 5 and appendices 1 and 2 detail some of the calculations used to arrive at the final results.

2. The formalism

2.1. Basic expressions

We consider a many-body system described by the Hamiltonian

$$H = H^0 + \lambda V - \boldsymbol{A} \cdot \boldsymbol{F}(t) \tag{1}$$

where H^0 is the largest part of H which can be diagonalised (analytically), λV is a binary-type interaction, assumed non-diagonal, and $-\boldsymbol{A} \cdot \boldsymbol{F}(t)$ is the external field Hamiltonian.

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We will use a representation in which H^0 is diagonal, indicated by d as subscript or superscript.

For conductivity problems, $\boldsymbol{F}(t) = q\boldsymbol{E}(t)$ and $\boldsymbol{A} = \Sigma(\boldsymbol{r}_i - \langle \boldsymbol{r}_i \rangle_{\text{eq}}) = \Sigma_1 \alpha_i$, q is the charge of the carriers (electrons), $\boldsymbol{E}$ is the electric field, and $\langle \boldsymbol{r}_i \rangle_{\text{eq}}$, r_i are the positions of the carriers before and after the application of the electric field. In the first Born approximation and for linear responses the diagonal part of the average current density reads [3]

$$\langle J_\mu \rangle_{\text{d}} \rangle_t = \frac{q}{\Omega} \sum_\zeta (\mathcal{B}_\zeta \langle n_\zeta \rangle_t \alpha_{\mu\zeta} + \langle n_\zeta \rangle_t \dot{\alpha}_{\mu\zeta}) \qquad \mu = x, y, z \tag{2}$$

where Ω is the volume of the system. Furthermore, $\alpha_{\mu\zeta} = (\zeta|\alpha_\mu|\zeta)$, $|\zeta)$ is the one-particle eigenstate of h^0 ($H^0 = \Sigma h^0$), $\langle n_\zeta \rangle_t$ is the average occupancy of the state $|\zeta)$ and $\mathcal{B}_\zeta \langle n_\zeta \rangle_t$ is the collision integral of the *quantum* Boltzmann equation for scattering between different particles (one-body collisions [3] or like particles (two-body collisions) [4]. The second term of (2) is the usual diffusive (or ponderomotive) current; the first term of (2), absent in a semiclassical treatment, represents many-body contributions of collisions to the current and has been termed 'collisional' current.

The non-diagonal part of the current density leads to an expression for $\sigma_{\mu\nu}^{\text{nd}}$ (cf reference [3], equation (3.21)) which is independent of the interaction. For the present problem it can be shown that σ_{xx}^{nd} and σ_{zz}^{nd} vanish identically; the component σ_{yx}^{nd} has been evaluated in [2].

The total conductivity is given by $\sigma_{\mu\nu} = \sigma_{\mu\nu}^{\text{d}} + \sigma_{\mu\nu}^{\text{nd}}$.

2.1.1. One-body collisions. If only the ponderomotive current exists, the DC conductivity tensor $\sigma_{\mu\nu}^{\text{d}}$, corresponding to (2) is given by (cf reference [3], equation (2.55))

$$\sigma_{\mu\nu}^{\text{d}} = -q^2/\Omega \sum_\zeta \frac{\partial \langle n_\zeta \rangle_{\text{eq}}}{\partial \varepsilon_\zeta} \frac{v_{\nu\zeta} v_{\mu\zeta}}{1\tau(\varepsilon_\zeta)^{-1}} \tag{3}$$

where $v_{\mu\zeta} = \dot{\alpha}_{\mu\zeta}$ and $\tau(\varepsilon_\zeta)$ is the relaxation time depending on the one-particle eigenvalue ε_ζ. If we have only 'collisional' current the DC conductivity tensor reads (cf reference [3], equation (2.84) for $\mu = \nu$).

$$\sigma = \beta q^2/2\Omega \sum_{\substack{\zeta,\zeta' \\ \text{spin}}} \langle n_\zeta \rangle_{\text{eq}} (1 - \langle n_{\zeta'} \rangle_{\text{eq}}) W_{\zeta\zeta'} (\alpha_{\mu\zeta} - \alpha_{\mu\zeta'})^2. \tag{4}$$

Here $\beta = 1/k_{\text{B}}T$, T is the temperature, and $W_{\zeta\zeta'}$ is the binary transition rate given by the Golden Rule. Equation (3) is valid for elastic or nearly elastic collisions, whereas formula (4) holds for both elastic and inelastic collisions.

When electrons interact with randomly distributed impurities (assumed to remain at equilibrium) the transition rate $W_{\zeta\zeta'}$, appearing in (3) and (4), is given by

$$W_{\zeta\zeta'} = \frac{2\pi}{\hbar} \frac{N_{\text{I}}}{\Omega} \sum_q |U(q)|^2 |(\zeta'|e^{i\boldsymbol{q}\cdot\boldsymbol{r}}|\zeta)|^2 \delta(\varepsilon_\zeta - \varepsilon_{\zeta'}) \tag{5}$$

where N_{I} is the impurity concentration, Ω is the volume, $U(\boldsymbol{q})$ is the Fourier transform of the impurity potential $U(\boldsymbol{r} - \boldsymbol{R})$, and $\boldsymbol{r}, \boldsymbol{R}$ are the positions of the electron and the impurity, respectively.

2.2. Superlattice in a magnetic field

We consider a superlattice which consists of n identical potential wells, of width d, separated by n identical barriers of width b and constant potential height $V(z) \equiv W$. A magnetic field $\boldsymbol{B}$ is applied along the superlattice direction ($\boldsymbol{B} = B\hat{\boldsymbol{z}}$). In the Landau gauge the one-electron Hamiltonian h^0, eigenstates $|\zeta)$, and eigenvalues ε_ζ read ($\boldsymbol{P}$ is the momentum operator).

$$h^0 = (\boldsymbol{P} + e\boldsymbol{A})^2/2m^* + W \qquad \boldsymbol{A} = (0, Bx, 0) \tag{6}$$

$$|\zeta) \equiv |N, k_y) \otimes |n, i, k_z) = \varphi_N(x - l^2 k_y)(e^{ik_y y}/\sqrt{L_y}) \otimes |n, i, k_z) \tag{7}$$

$$\varepsilon_\zeta \equiv \varepsilon_{N,i,k_z} = (N + \tfrac{1}{2})\hbar\omega_0 + \varepsilon_i(k_z) \quad N = 0, 1, \ldots, \quad i = 1, 2, \ldots \tag{8}$$

where $\omega_0 = |e|B/m^*$ is the cyclotron frequency, $l = (\hbar/m^*\omega_0)^{1/2}$ is the radius of the cyclotron orbit, $\boldsymbol{A}$ is the vector potential, and m^* is the effective mass in the well. Further, $\varphi_N(x)$ are the harmonic-oscillator wavefunctions, with Landau-level index N, and $|n, i, k_z)$ is the wavefunction in the z direction with eigenvalue $\varepsilon_i(k_z)$, i being the miniband index. The wavevectors in the y and z direction are k_y and k_z, respectively. We assume that each well and barrier has a width of the order of 100 Å and the total length of the superlattice, $L_z = n(d + b) = nc$, is much smaller than the other dimensions L_x and L_y. That is, the superlattice is a series of n quasi-two-dimensional wells separated by n quasi-two-dimensional barriers.

For the calculations of the following sections, we need the matrix element $(\zeta'|e^{i\boldsymbol{q}\cdot\boldsymbol{r}}|\zeta)$. Using (7) we find

$$|(\zeta'|e^{i\boldsymbol{q}\cdot\boldsymbol{r}}|\zeta)|^2 = |J_{NN'}(q_x, k_y, k_y')|^2 \delta_{k_y, k_y' + q_y} |(n', i', k_z'|e^{iq_z z}|n, i, k_z)|^2 \tag{9}$$

where

$$|J_{NN'}(\ldots)|^2 \equiv |J_{NN'}(u)|^2 = \frac{N!}{N'!} e^{-u} u^{N'-N} [L_N^{N'-N}(u)]^2 \qquad N \leqslant N' \tag{10}$$

here $u = l^2(q_x^2 + q_y^2)/2$, and $L_N^M(u)$ is a Laguerre polynomial.

3. Conduction parallel to the superlattice planes

3.1. The component σ_{xx}

We take the electric field along the x or y direction. As is well known, the diagonal (in the representation of (7)) velocity matrix elements which appear in (3) ($\nu = \mu = x$) vanish, so that the ponderomotive (or diffusive) contribution to the current is zero. We are left with (4) in which

$$\alpha_{\mu\zeta} = (\zeta|x|\zeta) = l^2 k_y. \tag{11}$$

Furthermore, the last factor in (9) is denoted by $I^{k_z}_{n\hat{n}',i}(q_z) \equiv |(n', i', k_z'|e^{iq_z z}|n, i, k_z)|^2$ where we have suppressed the dependence on i' and k_z' in view of the approximations that will follow.

We assume that the electrons are scattered quasi-elastically by randomly distributed impurities only within the wells where most of the electrons are found [5]. We write the impurity potential $U(\boldsymbol{r} - \boldsymbol{R})$, where $\mathbf{r}$ and $\mathbf{R}$ are the positions of the electron and of the impurity, respectively, as $\Sigma_{\mathbf{q}} U_{\mathbf{q}} e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$ in Fourier space. Since each well is quasi-two-dimensional we approximate U_q by its two-dimensional version $U_{q_\perp}(\boldsymbol{q}_\perp = q_x\hat{\boldsymbol{x}} + q_y\hat{\boldsymbol{y}})$.

3.1.1. Long-range potentials. For $U(r) = (e^2/\epsilon_0 r)\,e^{-k_s r}$, where ϵ_0 is the dielectric constant and k_s the inverse screening length, we have, in two dimensions, $U(q) = (2\pi e^2/\epsilon_0)(q_\perp^2 + k_s^2)^{-1/2}$. Now, due to the Kronecker delta in (9) the factor $(\alpha_{\mu\zeta} - \alpha_{\mu\zeta'})^2$ in (4) becomes, with the help of (11), $l^4 q_y^2$ for $\mu = x$ and $l^4 q_x^2$ for $\mu = y$. The remainder of the dependence of the conductivity on $q_\perp$ is the same in both cases. We can therefore use the symmetry $\sigma_{xx} = (\sigma_{xx} + \sigma_{yy})/2$ to replace $l^4 q_y^2$ by $2l^2 u$. Moreover, since we are considering elastic scattering and the inter-miniband separation is usually large, of the order of 20 meV for GaAs, we neglect any inter-miniband transitions allowed by the energy conserving δ-function, appearing in $W_{\zeta\zeta'}$, cf. equation (5). With all these details and $\Omega \to A_0 = L_x L_y$, in (4) and (5), we obtain

$$\sigma_{xx} \simeq \left(\frac{2\pi^2 \beta e^2 N_I l^2 U_0^2}{h A_0^2}\right) \sum_{\substack{N,N',n,n',i \\ q,k_y,k_z,k_z'}} u|J_{NN'}(u)|^2 I_{nn'i}^{k_z k_z'}(q_z)(q_\perp^2 + k_s^2)^{-1}$$

$$\times\, \delta[\varepsilon_{N',i,k_z'} - \varepsilon_{N,i,k_z}] f_{N,i,k_z}(1 - f_{N,i,k_z}), \tag{12}$$

where we put $\langle n_\zeta \rangle_{eq} = f_{N,i,k_z}$, the Fermi–Dirac function, and where $U_0 = 2\pi e^2/\epsilon_0$. Due to the exponential e^{-u}, in (10), the major contribution to the integral over $q_\perp (\Sigma_{q_\perp} \to (A_0/2\pi l^2)\int_0^\infty du)$ comes from small values of $u = l^2 q_\perp^2/2$. Therefore, for $q_\perp \ll k_s$ we can neglect the term $q_\perp^2$ in $(q_\perp^2 + k_s^2)^{-1}$. Alternatively, we can expand $(q_\perp^2 + k_s^2)^{-1}$ in powers of $q_\perp^2/k_s^2$, the leading contribution being $1/k_s^2$ for $q_\perp < k_s$. Also, as shown in appendix 1, $I_{nni}^{k_z}(q_z)$ is one or two orders of magnitude larger than $I_{n,n\pm1,i}^{k_z}(q_z)$ if the barriers are not too thin. Usually, the first miniband in (AlGaAs)GaAs is very narrow, a few meV, but not the second. Therefore for the first miniband it is reasonable to approximate the sum over k_z' by its value for $k_z' = k_z$; it is a poor approximation for the second miniband, but we make it in order to avoid excessive numerical work. Furthermore, the sum over k_y in (12), using $0 \leq l^2 k_y = x_0 \leq L_x$ gives a factor $A_0/2\pi l^2$. Taking spin into account we obtain from (12)

$$\sigma_{xx} \simeq \frac{e^2}{h}\beta\sigma_0 \sum_{\substack{N,N',i,n \\ q_z,k_z}} f_{N,i,k_z}(1 - f_{N,i,k_z}) I_{nni}^{k_z}(q_z)\delta[\varepsilon_{N'} - \varepsilon_N] \equiv \int_0^\infty u|J_{NN'}(u)|^2\,du \tag{13}$$

where $\sigma_0 = (N_I U_0^2/k_s^2 l^2)$ and $\varepsilon_N = (N + \frac{1}{2})\hbar\omega_0$. Since the wells are identical $I_{nni}^{k_z}(q_z)$ is independent of the well index, $I_{nni}^{k_z}(q_z) \equiv I_i^{k_z}(q_z)$. The integral over u is equal to $N + N' + 1$. As for the δ-function we replace it by a Lorentzian, of width Γ_i and of shift zero, and from the sum over N' we retain the largest contribution which occurs for $N' = N$. This is equivalent to neglecting, for elastic scattering, inter-Landau-level transitions. Then (13) gives

$$\sigma_{xx} \simeq \frac{e^2}{h}\frac{n\beta\sigma_0}{\pi} \sum_{N,i,k_z,q_z} (2N + 1) f_{N,i,k_z}(1 - f_{N,i,k_z}) I_i^{k_z}(q_z)/\Gamma_i. \tag{14}$$

Since the energy levels of an isolated well, of height W, fall within the minibands of the superlattice [6] it is a reasonable approximation, when the latter are narrow, to neglect the dependence of $I_i^{k_z}(q_z)$ on k_z and to evaluate $I_i(q_z)$ using the wavefunctions of the isolated well. Again, the approximation is good only for the lowest miniband. This is done in appendix 1 with account taken of the effective-mass difference between wells and barriers; then $I_i = \Sigma q_z I_i(q_z)$ is evaluated numerically. To proceed further we need to know the k_z-dispersion of the energy spectrum. We evaluate it numerically using

equation (3) of reference [7], which takes into account the effective-mass difference between wells and barriers. In many cases, depending on the values of the barrier width and height, the $\varepsilon_i(k_z)$ versus k_z dispersion relation can be well approximated by a straight line of slope λ_i: $\varepsilon_i(k_z) \simeq \varepsilon_i + \lambda_i k_z$, where ε_i is the bottom of the ith miniband (see references [1, 2] and [8]). We can then do the sum over k_z, in (14), analytically by transforming it into an integral. With $\Sigma_{k_z} \to 2(c/2\pi)\int_0^{k_0} \mathrm{d}k_z$, $k_0 = \pi/c$ ($\varepsilon_i(k_z) = \varepsilon_i(-k_z)$) and $\beta f_\zeta(1 - f_\zeta) = -\partial f/\partial \varepsilon_\zeta$ we obtain from (14) and (A2.1)

$$\sigma_{xx} \simeq \frac{ne^2}{\pi h} \sum_{N,i} \frac{\Gamma_i}{\lambda_i k_0} (2N+1)\{[1 + \mathrm{e}^{-\beta b_{Ni}}]^{-1} - [1 + \mathrm{e}^{\beta(\lambda_i k_0 - b_{Ni}}]^{-1}\} \tag{15}$$

where $b_{Ni} = \varepsilon_F - \varepsilon_i - (N + 1/2)\hbar\omega_0$ and $\lambda_i k_0$ is the bandwidth of the ith miniband. The quantity in the curly brackets has maxima for $b_{Ni} \to \lambda_i k_0/2$ and only the Nth level contributes appreciably when $\varepsilon_F - \varepsilon_i - \lambda_i k_0/2$ is at this level. For $b_{Ni} \gg \lambda_i k_0/2$ the same quantity tends to zero, i.e. we have well-defined Shubnikov–de Haas oscillations as observed, cf. references [1, 10, 11] with period $\hbar\omega_0$. The peak value of the conductivity obtained for $b_{Ni} \to \lambda_i k_0/2$ becomes

$$\sigma^p \approx \frac{e^2}{\pi h} n(2N+1) \sum_i \frac{\Gamma_i}{\lambda_i k_0} \tanh(\beta \lambda_i k_0/4). \tag{16}$$

For $(\beta\lambda_i k_0/4) \simeq 2$, σ^p is independent of temperature since $\tanh(\ldots) \to 1$. However, the minima of (15), occurring for $b_{Ni} \gg \lambda_i k_0/2$, show an activated behaviour, as observed [1], since $\{\ldots\} \simeq \mathrm{e}^{-\beta(b_{Ni} - \lambda_i k_0)}$ for $\beta b_{Ni} \gg 1$ and $b_{Ni} \gg \lambda_i k_0/2$.

The peak values σ^p of the conductivity, as given by (16), are in good agreement with the experimental values of [1]. For example, using the experimental values for the resistivities ρ_{xx} and ρ_{xy} in $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$ we obtain, at $B \simeq 6$ T, $\sigma^p \simeq 1.1 \times 10^{-3}$ Ω^{-1}. The first miniband width $\lambda_i k_0$ is 2.5 meV, $N = 1$, $n = 30$, and $T = 150$ mK. For the given [2] superlattice specifications we obtain from (A1.4) and (A2.1) $\Gamma_1 \simeq 2.6$ meV for $k_s l \simeq 5$. Equation (16) then gives $\sigma^p \simeq 1.35 \times 10^{-3}$ Ω^{-1} which is slightly above the experimental value.

To arrive at (15) we have used the approximation $\varepsilon_i(k_z) \simeq \varepsilon_i + \lambda_i k_z$ instead of the tight-binding expression $\varepsilon_i(k_z) = \bar{\varepsilon}_i + (\Delta_i/2)\cos k_z c$, where $\bar{\varepsilon}_i$ and Δ_i are the centre and the width of the ith miniband, respectively. We did so because the integral over k_z, in (14), is very cumbersome when the latter expression is used for finite temperatures. Near zero temperature, however, it can be done with the approximation $\beta f_\zeta(1 - f_\zeta) \simeq \delta(\varepsilon_\zeta - \varepsilon_F)$; the result for σ_{xx}, corresponding to (15) for $T \to 0$, is

$$\sigma_{xx} \simeq \frac{ne^2}{\pi h} \sum_{N,i} \frac{2\Gamma_i}{\pi\Delta_i} (2N+1)[1 - (2\bar{b}_{Ni}/\Delta_i)^2]^{-1/2} \tag{16'}$$

where $\bar{b}_{Ni} = \varepsilon_F - \bar{\varepsilon}_i - (N + \frac{1}{2})\hbar\omega_o$. Equation (16′) diverges for $b_{Ni} \to \bar{\Delta}_i/2$ showing clearly the oscillations of σ_{xx} with changing magnetic field. These divergences are removed when the δ-function is broadened but the result is too unwieldy to be given here. The advantage of (15) is that the approximate result for σ_{xx} is simple and valid for finite temperatures.

The maxima of (15) and (16′) occur when $\varepsilon_F - \varepsilon_i - \lambda_i k_0 = (N + \frac{1}{2})\hbar\omega_0$ and $\varepsilon_F - \bar{\varepsilon}_i - \Delta_i/2 = (N + \frac{1}{2})\hbar\omega_0$, respectively. Since the left-hand sides of these expressions change when the Fermi level passes through the minibands so does the period of the oscillations. This behaviour has been observed in [11].

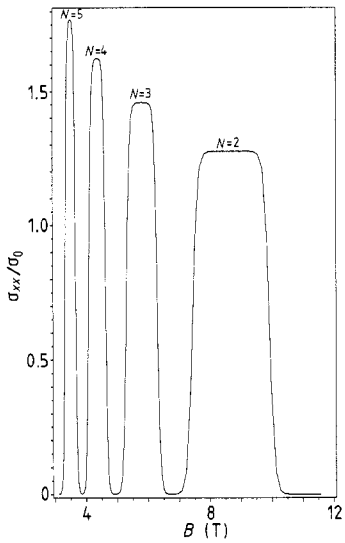

Figure 1. Magnetoconductivity σ_{xx}, in units of $\sigma_0 = e^2n/\pi h$, versus magnetic field for $i = 1$ at $T = 4$ K. N denotes the Landau levels.

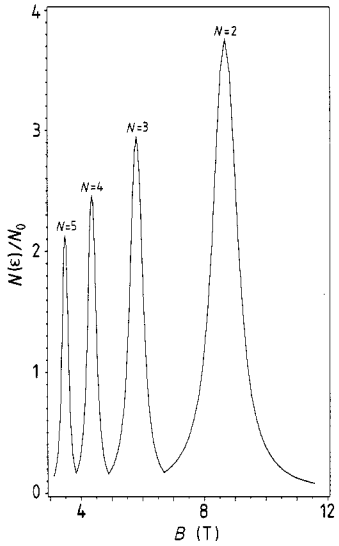

Figure 2. Density of states, in units of $N_0 = (A_0ne/\pi^2\hbar)$ Tesla versus magnetic field for $i = 1$. N denotes the Landau levels.

The oscillations of (15), as the magnetic field varies (i.e. as $b_{Ni} \to \lambda_i k_0/2$ changes) are also present in the density of states as a function of the magnetic field. Using the same approximation for the dispersion relation and a Lorentzian broadening of the δ-function we obtain

$$N(\varepsilon) = \sum_{N,n,i,k_y,k_z} \delta(\varepsilon - \varepsilon_i - \lambda_i k_z - (N + \tfrac{1}{2})\hbar\omega_0)$$

$$\simeq \frac{nA_0c}{\pi^2 l^2} \sum_{N,i} \Gamma_i \int_0^{k_0} \frac{\mathrm{d}k_z}{(b_{Ni} - \lambda_i k_0)^2 + \Gamma_i}$$

$$= N_0 \sum_{N,i} \{\tan^{-1}[(\lambda_i k_0 - b_{Ni})/\Gamma_i] + \tan^{-1}(b_{Ni}/\Gamma_i)\}/\lambda_i k_0 \qquad (17)$$

where $N_0 = nA_0/\pi^2 l^2$. Equation (17) shows maxima for $b_{Ni} \to \lambda_i k_0/2$, of value $2N_0 Ni \tan^{-1}(\lambda_i k_0/2\Gamma_i)\lambda_i k_0$ and minima for $b_{Ni} \gg \lambda_i k_0/2$, of value $\to 0$. The oscillations of the conductivity (equation (15)) and of the density of states (equation (17)) as the magnetic field varies, are shown in figures 1 and 2, respectively for $i = 1$. In either case only the Nth term of the sum over N contributes appreciably. The calculations are done numerically for (Al)GaAs–GaAs using the parameters of appendix A1 and equation (A2.1) for Γ_1.

The numerical results of [10, 11] for the density of states and the conductivity, without, e.g., the approximation $\varepsilon_i(k_z) \simeq \varepsilon_i + \lambda_i k_z$, show a similar oscillatory dependence on the magnetic field thus indicating that the approximation is reasonable. The same holds for the density of states [2, 15] and the thermopower [15] when one uses the tight-binding dispersion relation $\varepsilon_i(k_z) = \bar{\varepsilon}_i + (\Delta_i/2)\cos(k_z c)$. This behaviour is in line with the fact that the Fermi level ε_F, as determined (in general numerically) from the total number of electrons $N_e = \int N(\varepsilon) f(\varepsilon)\,\mathrm{d}\varepsilon$, oscillates as the magnetic field B varies.

For an analytical solution of this dependence, valid for finite temperatures, see reference [15].

3.1.2. Short-range potentials. For $U(\boldsymbol{r}) = U\delta(\boldsymbol{r})$, i.e., when the screening is complete, the results (12) to (17) can be taken over by replacing U_0/k_s by $\bar{U}_0$, where $\bar{U}_0$ is the Fourier transform (a constant, proportional to U) of the potential. Of course, in this case we do not have to make the approximation $q_\perp \ll k_s$.

3.2. The component σ_{yx}

For completeness we give the result for σ_{yx}. The calculation is reported in [2], in which only the lowest miniband, assumed to be very narrow, has been taken into consideration. If more minibands are occupied the result is easily obtained and reads (cf reference [2], equation (9) with $L_z \equiv 1$).

$$\sigma_{yx} = 2n(e^2/h) \sum_{N,i,k_z} f_{N,i,k_z}[1 + I_i^{k_z}(0)]. \tag{18}$$

For narrow minibands, $I_i^{k_z}(0)$, can be approximated by $I_i(0)$ as evaluated in appendix 1, cf. (A1.8), and is of order 10^{-5} or smaller [2]. Then, for a nearly linear dispersion relation, at finite temperatures, equation (18) gives, to order 10^{-5}

$$\sigma_{yx} = 2n(e^2/h)\left[i(N+1) + \sum_{N,i} \frac{1}{\beta\lambda_i k_0} \ln\left(\frac{1 + e^{\beta b_{Ni}}}{e^{\beta\lambda_i k_0} + e^{\beta b_{Ni}}}\right)\right]. \tag{19}$$

The first term of (19), obtained from (18) for $f_{N,i,k_z} \equiv 1$, is the result at zero temperature; the second term represents the finite temperatures corrections and vanishes as $\lambda_i \to 0$, i.e., as the limit of isolated quantum wells (with no energy dispersion in the z direction) is approached. We also notice that for $\exp(\beta b_{Ni}) \gg 1$ and $b_{Ni} > \lambda_i k_0$, the second term in (19) takes the form $-(N+1)\Sigma_i \exp[-\beta(b_{Ni} - \lambda_i k_0/2)]\,(\beta\lambda_i k_0)^{-1}$ and for $i = 1$ we have a simple activated behaviour for $\Delta\sigma_{yx}(T) = \sigma_{yx}(0) - \sigma_{yx}(T)$ as in the two-dimensional case [9].

4. Conduction along the superlattice: σ_{zz}

We take the electric field along the superlattice direction z. To use (3) we must first calculate $v_z = (\zeta|z|\zeta)$. For an infinitely deep quantum well $v_z = 0$ as in the case for an isolated well of finite weight W. Even in the limit of very thin barriers ('Kronig–Penney' model) v_z, which is independent of the magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{z}}$, is zero for the largest part of the first Brillouin zone [13]. Since we do not consider very thin barriers, we expect that v_z is close to zero, i.e. that the diffusion contribution to the current (if at all) expressed by (3) vanishes. Thus we are left with the 'collisional' contribution as expressed by (4), i.e., we consider only tunnelling (or hopping) of electrons, that are nearly localised in the wells, across the barriers when the electric field is applied.

We now remark that for conduction along the superlattice, i.e. through the barriers, the quantity $\alpha_{z\zeta} - \alpha_{z\zeta'}$, appearing in Eq. (4), is the mean distance involved in the transition $\zeta \to \zeta'$ ($W_{\zeta\zeta'}(\alpha_{z\zeta} - \alpha_{z\zeta'})$ is the mean probable distance per unit time that an

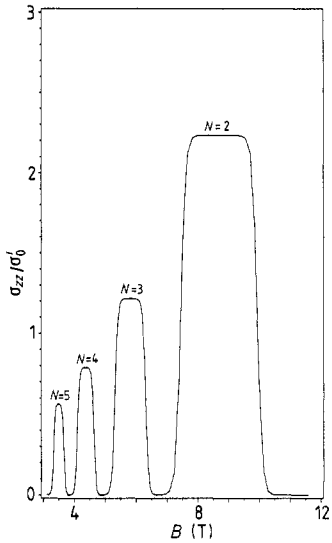

Figure 3. Magnetoconductivity σ_{zz}, in units of $\sigma_0' = (e^2n/\pi h)(c^2e/\hbar)$ Tesla, versus magnetic field for $i = 1$ at $T = 4$ K. N denotes the Landau levels.

electron travels along the z direction). Since the mean abscissa $z_{n,i,k}$ is the centre of the nth well in the absence of the electric field, we have

$$\left.\begin{aligned} &z_{n,i',k_z} - z_{n,i,k_z} = 0 \\ &z_{n\pm1,i',k_z'} - z_{n,i,k_z} = \pm(d+b) = \pm c \end{aligned}\right\} \text{all } n, i, k_z, i', k_z'. \tag{20}$$

We will assume that the barrier widths and heights are such that the transition probabilities $W_{\xi\xi'}$ are different from zero only for n, n' corresponding to neighbouring wells and zero otherwise. This is supported by the values of $I^{k_z}_{n,n\pm1,i}(q_z)$ for narrow minibands, as shown in appendix 1.

The calculation of the conductivity σ_{zz} proceeds almost exactly as in § 3. Instead of (11) we use (20); corresponding to (13) we now obtain

$$\sigma_{zz} \simeq \frac{e^2}{h}\frac{\beta\sigma_0}{2}\frac{c^2}{l^2}\sum_{\substack{N,N'i,n\\ q_z,k_z}} f_{N,i,k_z}(1-f_{N,i,k_z}) \times I^{k_z}_{n,n+1,i}(q_z)\delta[\varepsilon_{N'} - \varepsilon_N]\int_0^\infty |J_{NN'}(u)|^2\,\mathrm{d}u. \tag{21}$$

The integral over u is equal to 1 and the quantity $I^{k_z}_{n,n+1,i}(q_z)$ denoted by $I'^{k_z}_i(q_z)$ and evaluated in appendix 1, is independent of n if we neglect boundary effects. The rest of the approximations are the same as those following (13). Corresponding to (14) for σ_{xx} we obtain

$$\sigma_{zz} \simeq \frac{e^2}{h}\frac{n\beta\sigma_0 c^2}{2\pi l^2}\sum_{N,i,k_z,q_z} f_{N,i,k_z}(1-f_{N,i,k_z})I'^{k_z}_i(q_z)/\Gamma_i' \tag{22}$$

and in (15) and (16) we have to replace $(2N+1)\Gamma_i$ by $(c^2/2l^2)\Gamma_i'$ to get the corresponding result for σ_{zz}. Therefore, σ_{zz} oscillates as function of the magnetic field and with the same period as σ_{xx}, i.e., $\hbar\omega_0$. This is illustrated in figure 3, where σ_{zz} (cf equation (15)

with $(2N+1)\Gamma_1 \rightarrow (c^2/2l^2)\Gamma_1'$ is plotted versus B. It is interesting to take the ratio σ_{xx}/σ_{zz} for $i=1$. From (15) or (16), for σ_{xx}, and the corresponding equations for σ_{zz} we obtain, if we consider only the dominant contribution to the sum over N

$$r \equiv \sigma_{xx}/\sigma_{zz} \simeq 2(2N+1)(l^2/c^2)(\Gamma_1/\Gamma_1'). \tag{23}$$

The precise value of r depends on the magnetic field and the superlattice parameters; it increases with decreasing magnetic field (due to the factor 1 in $(2N+1)$ since $N \propto 1/B$, cf. references [1] and [10]. This is due to the fact that as B becomes weaker σ_{zz} decreases, cf. equation (15) with $(2N+1)\Gamma_1 \rightarrow (c^2/2l^2)\Gamma_1'$. This behaviour of σ_{zz} with B is similar to that obtained before for a three-dimensional sample [14]. Also, that r increases with decreasing B is in line with the expectation that the carriers acquire more freedom in the x direction as B becomes weaker.

5. Concluding remarks

We have shown analytically that well-defined Shubnikov–de Haas oscillations exist in superlattices with narrow minibands, whether the electric field is parallel to the layers or perpendicular to them, when a magnetic field is applied perpendicular to the layers. The results are quite simple and transparent and, to our knowledge, new at least in the form given here. Moreover, they are in reasonable agreement with the available experimental data. This is particularly true for (i) the period of the oscillations, (ii) the order of magnitude, and (iii) the activated behaviour of σ_{xx}, for which some data exist [1]. For want of data we cannot check points (ii) and (iii) for σ_{zz} although we expect a similar qualitative agreement. The comparison of σ_{yx} with the available experimental data [1] was made in reference [2].

It is perhaps of interest to point out the difference between transport parallel to and normal to the layers. In the former case, we have, for elastic impurity scattering, intra-Landau-level collisions whereas in the latter we have elastic tunnelling across a barrier. As expected and as reflected by (23), for magnetic fields such that $l \simeq c$, the conductivity parallel to the layers is much larger than that along the superlattice axis if the barriers are not too thin, since $\Gamma_1' \ll \Gamma$, cf appendices 1 and 2.

For the results of this paper we have relied extensively on some characteristics of the isolated quantum well with the same energy barrier as the superlattice and of two quantum wells separated by the superlattice barrier width. Apart from being instructive, this was done in order to avoid excessive numerical work. Therefore part of any disagreement with the experiments is to be ascribed to the difference between these characteristics and the ones pertaining to a superlattice.

A second limitation of the present results is that they apply only for weak electric fields since we used linear response theory. This allowed us to neglect the tilting of wells and barriers and thus to avoid further numerical work.

We have not considered at all the case of the magnetic field parallel to the layers. Although much more interesting, this case is much more complicated and one has to perform extensive numerical calculations [8].

Despite the above limitations, the results appear to be in reasonable agreement with the available experimental data. Obvious refinements as well as consideration of other interactions (e.g. electron–phonon interaction) and other effects (e.g. cyclotron-resonance) are left for future work.

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Appendix 1

Below we outline the calculation of the quantities $I^{k_z}_{nni}(q_z) = I^{k_z}_i)(q_z)$, and $I^{k_z}_{n,n+1,i}(q_z) = I'^{k_z}_i(q_z)$ which appear in the text, cf equations (13)–(16) and equations (21)–(22). We use for the calculation the wavefunctions of an isolated well, of width d and energy depth W, modifying slightly the procedure of reference [13] to take account of the effective-mass difference between wells and barriers. The normalised wavefunction is written as

$$\psi_i(z) = \begin{cases} C_i \sin\delta_i \exp[x_i(z - z_1)] & z < z_1 \\ C_i \sin[k_i(z - z_1) + \delta_i] & z < z < z_2 \\ (-1)^{i+1} \sin\delta_i \exp[-x_i(z - z_2)] & z > z_2 \end{cases} \tag{A1.1}$$

where $x_i = [2m^*_b(W - \varepsilon_i)]^{1/2}/\hbar$, $k_i = (2m^*_w\varepsilon_i)^{1/2}/\hbar$, $C_i^2 = (d/2 + 1/x_i)^{-1}$ and $\delta_i = \tan^{-1}(m^*_b k_i/m^*_w x_i)$. The effective masses in the well and barrier are m^*_w and m^*_b, respectively. The eigenvalues ε_i are given by

$$k_i d + 2\delta_i = i\pi \qquad i = 1, 2, \ldots . \tag{A1.2}$$

To obtain (A1.2) we used the continuity of the wavefunction and of the quantity $(1/m^*)\mathrm{d}\psi/\mathrm{d}z$ at the boundaries z_1 and z_2.

The quantity $I_i = \Sigma_{q_z} I_i(q_z) \to (c/2\pi)\int i_i(q_z)\,\mathrm{d}q_z$, appearing in (A2.1), (15) and (16), is given by the last factor in (9) for $i' = i$ and $n' = n$, upon neglecting its k_z dependence

$$I_i \simeq (c/2\pi)\int |(n,i)|\, \mathrm{e}^{\mathrm{i}q_z z}\, |n,i)|^2\, \mathrm{d}q_z. \tag{A1.3}$$

Since the result must be independent of the well index n we use (A1.1) and Parseval's theorem to obtain ($c = d + b$)

$$\begin{aligned} I_i &\simeq (c/2\pi)\int |\psi_i^*(z)\psi_i(z)|^2\, \mathrm{d}z \qquad -c/2 \leq z \leq c/2 \\ &= (cC_i^4/4\pi)\{(1 - \mathrm{e}^{-2x_i b})\sin^4\delta_i/x_i \\ &\quad + [3k_i d + \sin(2\delta_i)(4 - \cos 2\delta_i)]/4k_i\} \end{aligned} \tag{A1.4}$$

The only dependence of I_i on the barrier width b is through c and $1 - \exp(-2x_i b) \simeq 1$ for $x_i b > 1$. Using the parameters given after (A1.9) (see below) we obtain $I_1 \simeq 2.5 \times 10^{-1}$.

We now turn to the evaluation of $I'_i(q_z)$ and of $I'_i = \Sigma_{q_z} I'_i(q_z)$. Taking the origin at the centre of a barrier (of width b) separating the nth and the $(n + 1)$th wells we remark that

$$\psi_i^{n+1}(z) = (-1)^{i-1}\psi_i^n(-z). \tag{A1.5}$$

It follows that

$$\bar{I}_i(q_z) = \int_{-\infty}^{\infty} [\psi_i^n(z)]^*\, \mathrm{e}^{\mathrm{i}q_z z}\, \psi_i^{n+1}(z)\, \mathrm{d}z = (-1)^{i+1} J_i(-q_z) + J_i(q_z) \tag{A1.6}$$

with

$$J_i(q_z) = \int_0^\infty [\psi_i^n(z)]^* \, e^{iq_z z} \psi_i^{n+1}(z) \, dz. \tag{A1.7}$$

The integration over z, which includes the barrier region in conjunction with (A1.5), is done analytically but the result is too lengthy to be given here. A relatively simple result is obtained upon neglecting terms proportional to $\exp(-x_i c)$ in front of terms varying like $\exp(-x_i b)$. With $q_z \equiv q$ we obtain

$$I_i'(q) \equiv |\bar{I}_i(q)|^2 \simeq 4C_i^4 \, e^{-2x_i b} \sin^4 \delta_i \times \left(\frac{\sin L}{q} + \frac{2x_i^2(x_i^2 + k_i^2) \cos L - q(3x_i^2 - k_i^2)q^2) \sin L}{(x_i^2 + k_i^2 - q^2)^2 + 4q^2 x_i^2} \right)^2 \tag{A1.8}$$

where $L = qb/2$. For $q \to 0$ we get the square of the overlap integral between neighbouring wells as

$$I_i'(q) \xrightarrow{q \to 0} 4C_i^4 \sin(4\delta_i) \, e^{-2x_i b} \left(\frac{b}{2} + \frac{2x_i}{x_i^2 + k_i^2} \right)^2. \tag{A1.9}$$

As expected, both (A1.8) and (A1.9) fall exponentially with increasing barrier width b. The final integration over $q_z = q$ is done numerically using not (A1.8) but the exact result for $I_i'(q)$. For $d = 188$ Å, $b = 40$ Å, $W = 100$ meV, $m_w^* = 0.67m_0$, $m_b^* \simeq 0.092m_0$ we obtain $(I_i' = \Sigma_{q_z} I_i'(q_z))$, $I_1' \sim 1 \times 10^{-3}$, $I_2' \sim 2.25 \times 10^{-2}$, $I_3' \sim 2.3 \times 10^{-1}$. Notice that I_1' is about two orders of magnitude smaller than I_1, cf (A1.4).

Appendix 2

The level widths Γ_i, Γ_i' which appear in the text are estimated from the scattering rate: $\Gamma_\zeta \simeq \hbar/\tau_\zeta$ with $1/\tau_\zeta = \Sigma_{\zeta'} W_{\zeta\zeta'}$. We use (5), (9) and (10). In (9) we neglect the dependence on k_z and we take $i = i'$. For Γ_i we take $n' = n$ and for Γ_i', $n' = n + 1$ corresponding to conduction normal to and along the superlattice direction, respectively. For the screened interaction $U(r) = (e^2/\epsilon_0 r) \exp(-k_s r)$ we neglect $q_\perp^2$ in front of k_s^2. Keeping only the largest contribution from the sum over N' and k_z' (i.e. $N' = N$ and $k_z' = k_z$ only) and replacing the δ-function in (5) by a Lorentzian we obtain

$$\Gamma_i \simeq [I_i N_I U_0^2/\pi l^2 k_s^2]^{1/2} = [I_i \sigma_0]^{1/2}. \tag{A2.1}$$

$\Gamma_\zeta \simeq \hbar/\tau_\zeta$ with $1/\tau_\zeta = \Sigma_{\zeta'} W_{\zeta\zeta'}$. We use (5), (9) and (10). In (9) we neglect the depen- k_s must be replaced by U_0 where U_0 (=constant) is the Fourier transform of the potential.

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